Wave Digital Approach – A Theoretical Model of Step Discontinuity

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Abstract – **A theoretical model for the modeling of the microstrip structure as well as one type of regular discontinuity (step) is described here. Wave digital network is a model of the microstrip structure modeled by wave digital elements. A section number in the model has direct influence on the sampling frequency of that digital model, and on accuracy of the desired response. Also, it is very important to achieve a good compensation of the effects of the identified discontinuities. A new approach of modeling of the equivalent *L* discontinuity network given here involves increasing lengths of the lines in junction. One test example, proving the response accuracy is given.**

Keywords – **Wave digital approach, step discontinuity, transmission lines, microstrip circuits**

I. Introduction

Modeling of the planar structures by wave digital elements, based on well known theory of wave digital filters [1]-[3], can be efficiently used for analysis of these structures in both the time and the frequency domains. Microwave planar structures can be modeled by one-dimensional [4]-[6] and by two-dimensional [7] wave digital elements.

A nonuniform structure has to be divided into cascade-connected uniform transmission lines (*UTL*). A lossless uniform transmission line is modeled by a two-port digital element with a delay appears in forward path. This wave digital two-port is called the unit element (*UE*) [2]. The port resistances of the *UE* are equal and correspond to the characteristic impedance of *UTL*. The connection of two *UE* with different port resistances is achieved by two-port adaptors (*TA*), [3].

In the complex microstrip structures, delays of the transmission lines vary from one another and because of this, each transmission line has to be represented as a cascade connection of a certain number of *UE*. A way of determination a minimal section numbers in wave digital network (*WDN*) of complex microstrip structure is given in the papers [8]-[9]. Number of section in *WDN* has direct influence on the sampling frequency of that digital model, and on accuracy of desired response.

Efficient and very simple algorithms for calculating transmission and input reflection coefficients of the wave digital networks are described in the papers [5]-[6]. The algorithms are very easily implemented in the *MATLAB* environment. The analysis of the wave digital structure is efficiently automated, which is inevitable when structure with large numbers of building blocks (*UE* and *TA*) are to be dealt with.

In the previously published papers [4]-[6] and [8], nonuniform microstrip structures are observed as cascaded *UTL* segments, but effects of the step discontinuities have not been taken under consideration. But, once the step discontinuities have been identified in the structure, they must be corrected. One wave digital model of the asymmetrical equivalent *T*-network of this discontinuity is described in the paper [10]. In the paper [11], two new approaches of modeling equivalent circuit of this discontinuity, based on the approximation of the reactive elements by transmission lines, are given. Here, the lumped and distributed element approximate equivalent relationships are also used, but a new approach of modeling step discontinuity is shown.

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II. Approximate Equivalent Relationships

From the electric circuit theory it is well known, that lumped elements (an inductance in series branch and a capacitance in parallel branch) can be approximated by two-port transmission lines, [10]-[11]. For high frequencies, an inductance is replaced by transmission line of high characteristic impedance (Fig.1), and a capacitance by transmission line of low characteristic impedance (Fig.2).

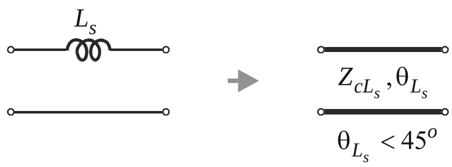

Fig.1. Aproximation of an inductance in series branch by transmission line of high characteristic impedance Z_{cL_s} .

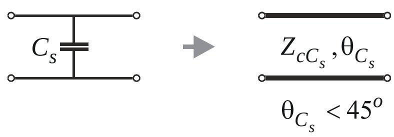

Fig.2. Aproximation of a capacitance in parallel branch by transmission line of low characteristic impedance Z_{cC_s} .

An electrical length of a transmission line corresponding to the capacitance C_S can be found as

$$\theta_{C_s} = 2\pi f_C \cdot C_S \cdot Z_{cC_s} \; . \tag{1}$$

Also, an electrical length of a transmission line corresponding to the inductance L_S can be found as

$$\theta_{L_s} = 2\pi f_C \cdot L_S \,/\, Z_{cL_s} \; . \tag{2}$$

Parameter f_c is the cuttoff frequency for lowpass and highpass filters and the center frequency for bandpass and bandstop filters.

III. Modeling of the Step Discontinuity Equivalent Network

All practical microwave circuits, even the simplest, contain some type of discontinuities. A discontinuity in the width of a microstrip line, Fig.3*a*, is a very often used in the microstrip circuits in order to change characteristic impedance of the line. This is important for design of the filters and the impedance matching networks.

This discontinuity is typically modeled by using quasistatic analysis, which enables their lumped parameters, namely, L_s and C_s, to be derived. In practice, accuracy of the discontinuity models depends on their physical dimensions [12].

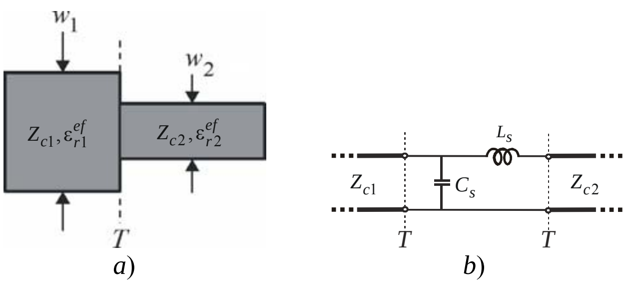

Fig.3. *a*) Step discontinuity,
b) Equivalent network of the discontinuity.

Equivalent network of the discontinuity has a parallel capacitance C_s placed on the side of the wide line, and the series inductance L_s placed on the side of the narrow line, as shown in Fig.3*b*.

The simplest modeling approach of the equivalent discontinuity network involves changing line lengths. In order to absorbe discontinuity effects a new line lengts are counted. A capacitor in parallel branch has the same effect as increasing of the wide line length for

$$d_{C_s} = c \cdot C_s \cdot Z_{cC_s} / \sqrt{\varepsilon_{r1}^{ef}} \,, \tag{3}$$

where ε_{r1}^{ef} is effective dielectric constant of the wide line in the structure shown in Fig.3*a*. An inductor in series branch has the same effect as increasing of the narrow line length for

$$d_{L_s} = c \cdot L_s / \left(Z_{cL_s} \cdot \sqrt{\varepsilon_{r2}^{ef}} \right), \tag{4}$$

where ε_{r2}^{ef} is effective dielectric constant of the narrow line in the structure shown in Fig.3*a*. The physical lengths of the transmission lines depend on their characteristic impedances which can be chosen as follows:

- *In the first case*, characteristic impedances are found to be equal with the characteristic impedances of their neighbour lines. Impedance of the transmission line used for aproximation of the series inductance is equal to the high characteristic impedance of the narrow line in junction. Characteristic impedance of the transmission line used for aproximation of the parallel capacitance is equal to the low impedance of the wide line in junction.

- *In the second case*, characteristic impedances are chosen as follows: impedance of the transmission line used for aproximation of the series inductance is chosen to be $Z_{cL_s} = 150\,\Omega$, and impedance of the transmission line used for aproximation of the parallel capacitance is $Z_{cC_s} = 5\,\Omega$. This is a result of the well known fact that typical values of these impedances are $\leq 10\,\Omega$ and $\geq 150\,\Omega$, respectively.

IV. Wave Digital Network

Planar microstrip structure shown in Fig.3*a*, can be aproximated by a cascade connection of two transmission lines as shown in Fig.4. Each *UTL* segment from Figure 3*a* is aproximated by one transmission line (Z_{c1} and Z_{c2}). The equivalent network of the discontinuity is included through increasing lengths of the transmission lines.

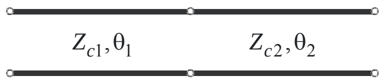

Fig.4. Cascade connection of two transmission lines.

Then, each transmision line is modeled by certain number of *UE*. A nonuniform planar microstrip structure shown in Fig.3*a*, is modeled by a wave digital network (*WDN*) composed only of two types of building blocks (*UE* and *TA*) as shown in Fig.5. This type of *WDN* is analyzed very easily and simply by using wave transfer matrices and algorithms given in the papers [5]-[6].

Adaptor coefficients in the *WDN* are

$$\begin{aligned} \alpha_G &= (R_g - Z_{c1})/(R_g + Z_{c1}), \\ \alpha_{V12} &= (Z_{c1} - Z_{c2})/(Z_{c2} + Z_{c1}), \\ \alpha_P &= (Z_{c2} - R_p)/(Z_{c2} + R_p). \end{aligned} \tag{5}$$

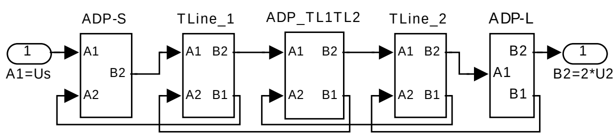

Fig.5. *WDN* of the structure shown in Fig.3
with modeled discontinuity elements.

In the other modeling approaches of the step discontinuity, new blocks have to be included in the *WDN*, as described in the papers [9]-[11]. This approach of modeling step discontinuity is very acceptable because there are not new blocks here. In the case of cascaded microstrip lines, the proposed method reduces drastically the computation time while giving acceptable accuracy.

V. Analysis Example

A microstrip stepped-impedance 7[th] order Chebyshev lowpass filter with passband ripple of $0.0137\,dB$ and cutoff frequency of $900\,MHz$ [13] is used for verification of the proposed method. The layout is shown in Fig.6, and this circuit is also analyzed in the paper [5]. The effects of the step discontinuities have not been taken under consideration in that paper.

Fig.6. Layout of lowpass filter

Here, the microstrip lowpass filter is considered as a cascade connection of seven *UTL* segments. Their delays vary from one another because of their dependence on the effective dielectric constant, Table 1. In order to have delays in the wave digital models as possible equal to these delays, each transmission line has to be represented as a cascade connection of a certain number of *UE*.

WDN of this lowpass filter has the structure given in Figure 5, where a number of blocks corresponds to transmission lines is 7 and a number of blocks corresponds to two-port adaptors is 8.

TABLE 1: TRANSMISSION LINE PARAMETERS WITHOUT MODELED DISCONTINUITES

nv	d [mm]	Zc [Ohm]	Tv [ps]
1	18.1149	68.8833	122.1264
2	7.8345	15.9611	58.7383
3	32.0420	68.8833	216.9616
4	9.7713	15.9611	73.2599
5	32.0420	68.8833	216.9616
6	7.8345	15.9611	58.7383
7	18.1149	68.8833	122.6589

A minimal number of sections for the given error, can be found as described in the papers [8]-[9] where multiple factor $q \geq 1$ is used. For given error $n\_er = 0.01\,\%$, first relative error of delay with absolute value less then given error is for $q = 32$. A total minimal number of sections in *WDN* is $n_t = \sum_{k=1}^{7} n_k = 474$. For individual *UTL* segments, a number of sections n_k is 67, 32, 118, 40, 118, 32 and 67, respectively. A total delay for the digital model of the structure is $T_t = n_t \cdot T_{\min} / q = 870.0618\ ps$ where $T_{\min} = \min\{T_1, T_2, ..., T_7\} = 58.7383\,ps$ is a minimum delay. A total real delay of the structure is $T_\Sigma = \sum_{k=1}^{7} T_k = 869.9778\ ps$. A sampling frequency of the digital model of the planar structure for the chosen minimal number of sections is $F_s = n_t / T_t = 544.7889\,GHz$. A relative error of delay is $er = \dfrac{T_\Sigma - T_t}{T_\Sigma} \cdot 100\% = -0.0096\,\%$.

A. Line Characteristic Impedances chosen in the First Case

The characteristic impedances are found to be equal with the characteristic impedances of their neighbour lines, $Z_{cL_s} = 68.8833\,\Omega$ and $Z_{cC_s} = 15.9611\,\Omega$. Physical lengths of the transmission lines according to the relations (3) and (4) are: $d_{Cs} = 0.2943\,mm$ and $d_{Ls} = 0.4353\,mm$. Compensation of the discontinuity effects is done as foollows: a physical length of the transmisison line used for approximation of the narrow line is increased for value d_{Ls}, and a length of the transmission line corresponding to the wide line is increased for value d_{Cs}.

Transmission line parameters for this case are given in the Table 2. According to the parameters given in the Tables 1 and 2, it can be concluded that the number of transmission line is the same in both cases, but their physical length differ one another and beacuse their delays differ either.

TABLE 2: TRANSMISSION LINE PARAMETERS WITH MODELED DISCONTINUITES (69/16 IMPEDANCES)

nv	d [mm]	Zc [Ohm]	Tv [ps]
1	18.5502	68.8833	125.6066
2	8.4231	15.9611	63.1519
3	32.9127	68.8833	222.8570
4	10.3600	15.9611	77.6735
5	32.9127	68.8833	222.8570
6	8.4231	15.9611	63.1519
7	18.5502	68.8833	125.6066

For given error $n\_er = 0.01\,\%$, a total minimal number of sections in *WDN* is $n_t = 214$. For individual *UTL* segments, a number of sections n_k is 30, 15, 53, 18, 53, 15 and 30, respectively. A total delay for the digital model of the structure is $T_t = 900.9676\ ps$. A total real delay of the structure is $T_\Sigma = 900.9046\ ps$. A sampling frequency of the digital model of the planar structure for the chosen minimal number of sections is $F_s = 237.5224\,GHz$. A relative error of delay is $er = -0.0070\,\%$.

B. Line Characteristic Impedances chosen in the Second Case

The characteristic impedances are chosen to be: $Z_{cL_s} = 150\,\Omega$ and $Z_{cC_s} = 5\,\Omega$. Physical lengths of the transmission lines according to the relations (3) and (4) are: $d_{Cs} = 0.0922\,mm$ and $d_{Ls} = 0.1999\,mm$. In the Table 3, transmission line parameters are shown.

TABLE 3: TRANSMISSION LINE PARAMETERS WITH MODELED DISCONTINUITIES (150/5 IMPEDANCES)

nv	d [mm]	Zc [Ohm]	Tv [ps]
1	18.3148	68.8833	124.0126
2	8.0189	15.9611	60.1210
3	32.4418	68.8833	219.6689
4	9.9557	15.9611	74.6425
5	32.4418	68.8833	219.6689
6	8.0189	15.9611	60.1210
7	18.3148	68.8833	124.0126

For given error $n\_er = 0.01\,\%$, a total minimal number of sections in *WDN* is $n_t = 631$. For individual *UTL* segments, a number of sections n_k is 89, 43, 157, 53, 157, 43 and 89, respectively. A total delay for the digital model of the structure is $T_t = 882.2400\ ps$. A total real delay of the structure is $T_\Sigma = 882.2474\ ps$. A sampling frequency of the digital model of the planar structure for the chosen minimal number of sections is $F_S = 715.2248\,GHz$. A relative error of delay is $er = 0.0009\,\%$.

C. Results Comparison

A response comparison for the proposed approach of modeling step discontinuity when characteristic impedances are chosen in two different ways is shown in Fig. 7 and 8.

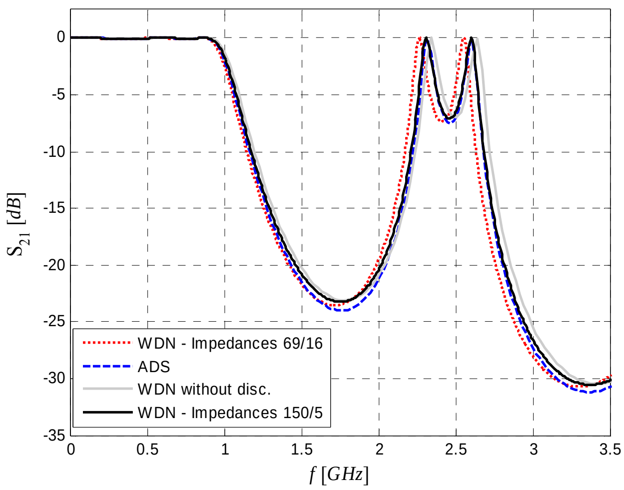

Fig.7. Response comparison.

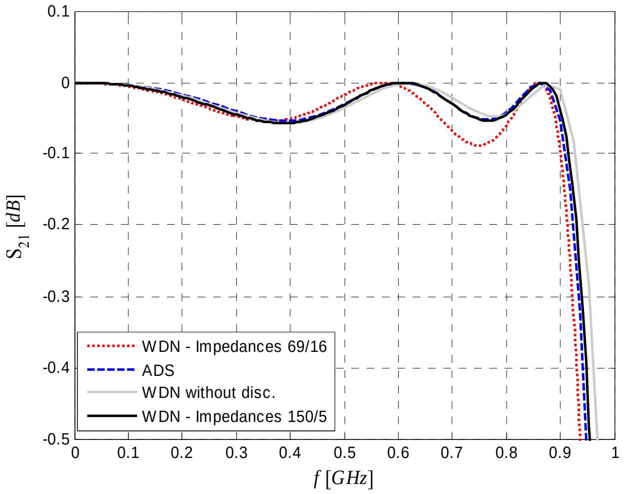

Fig.8. Comparison of the bandpass responses.

The results obtained by modeling structure and its discontinuities in the described approach are compared to those obtained in *ADS* (Advanced Design Systems). In the case of chosen characteristic impedances $Z_{cL_s} = 150\,\Omega$ and $Z_{cC_s} = 5\,\Omega$ (Approach *B*), the obtained result match very well with that one obtained in *ADS*. In the region below the frequency of $1.5\,GHz$, the agreement between the results is very good.

The curve corresponding to the characteristic impendances $Z_{cL_s} = 68.8833\,\Omega$ and $Z_{cC_s} = 15.9611\,\Omega$ (Approach *A*), differ slighty from that one obtained in *ADS*.

A curve for *WDN* without modeled discontinuities is shifted to the right in whole frequency band.

VI. Conclusion

In order to prove the accuracy of the proposed modeling approach of the step discontinuity, the computer simulated results obtained by *WDN* are compared to those obtained in *ADS*. One can observe that results obtained by described approach of modeling step discontinuity are in a very good agreement with *ADS* data in whole frequency band. If two cases for different choice of impedances (69/16 and 150/5) are compared, it can be seen that the approach of modeling discontinuities with 150/5 chosen impedances is better because those are typical impedance values for approximation of lumped elements by transmission lines.

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