Chebyshev IIR filter sharpening implemented on FPGA

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Abstract **— Filter design technique called filter sharpening (a new filter created from an existing filter in which the passband ripple is reduced and the stopband attenuation is increased) is presented. The proposed method is applied on the Chebyshev infinite impulse response (IIR) filter whereas in the past the 30 year old technique had been applied only to FIR filters with constant group delay. Using a small area of field programmable gate array chips for placing only two fourth-order IIR filters, and the hardware folding techniques, a number of different filter specifications can be fulfilled. The method is attractive for filter designs without extensive analysis of nonlinear effects and for the fixed point implementations.**

Keywords **— Chebyshev filter, filter sharpening, Field Programmable Gate Array – FPGA, filter design, Infinite Impulse Response IIR filter**

I. INTRODUCTION

THE operating rate of field programmable gate array (FPGA) chips can be considerably larger than the processing rate for digital signal processing (DSP) applications in audio, communication or control systems. The used area of FPGA chips can be significantly reduced by time sharing the hardware resources [1], that is, the same implemented filter structure is used several times between two successive input samples. An excellent solution is the hardware folding technique to time-multiplex many algorithm operations onto a single functional unit such as an adder or a multiplier [1]. Although this technique can be used for the implementation of infinite impulse response (IIR) filters as cascade connection of the second-order section (SOS) filters, the design by an inexperienced user may fail to fulfill the filter specification due to nonlinear effects such as overflow and dead-band effects, pure dynamic range of SOS filters, or coefficient quantization.

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Filter sharpening is a known technique in filter theory - a new filter is created from the existing low-order filter and is sharpened in such a way that the passband ripple is reduced and the stopband attenuation is increased [2]. This technique is not applicable to filters having non-constant group delay such as minimum phase finite impulse response (FIR) filters or IIR filters.

Digital filter design becomes an effortless task because it can be accomplished using powerful software tools. Unfortunately, many designs of selective IIR filters can fail to fulfill specifications in fixed point implementations with FPGA chips. The filter design can be simplified if the same kind of universal filter sections can be used as building blocks, rather than design for each set of specifications and perform thorough analysis of nonlinear effects.

In this paper we present a design method that consists of a cascade connection of several low-order IIR filter sections. The attenuation in the passband can be kept very low, while increasing the number of repeated sections increases the stopband attenuation. The advantage of an FPGA implementation of the proposed filter sharpening method is illustrated by an example.

II. EQUIRIPPLE RATIONAL FUNCTION

A. Rational magnitude function

It is easy to establish [3] the equiripple character of rational magnitude functions

$$\left|R(x)\right|^2 = \frac{1+\varepsilon_N^2 T_r^2(x)}{\left(1+\varepsilon_D^2 T_r^2(x)\right)^{\mu}} \qquad (1)$$

with the constraint

$$\varepsilon_N^2 = \left(1+\varepsilon_D^2\right)^{\mu} - 1 \qquad (2)$$

In these equations $T_r(x)$ is the *r*-th order Chebyshev polynomial. The constants ε_N and ε_D are called the ripple factors while the integer μ can be of any value $\mu \geq 2$.

B. Minima and maxima of the magnitude function

The examination of the first and the second derivatives of equation (1) show that the independent variables of the minima $x_{\min}$ of the rational function $\left|R(x)\right|$ are determined by the roots of the equations

$$T_r(x) = 0, \quad T_r^{'}(x) \qquad (3)$$

At $x_{\min}$ and $x = 1$ the value of the magnitude function is 1:

$$|R(x_{\min})| = 1, \quad |R(1)| = 1 \tag{4}$$

The position of maximum can be found from

$$T_r^2(x_{\max}) - k = 0 \tag{5}$$

$$k = \frac{\varepsilon_N^2 - \mu\varepsilon_D^2}{\varepsilon_N^2 \varepsilon_D^2 (\mu - 1)}, \quad \mu = 2,3,4,... \tag{6}$$

The ripple factor (the maximal variation of the magnitude function)

$$\varepsilon^2 = |R(x_{\max})|^2 - 1 \tag{7}$$

becomes

$$\varepsilon^2 = \frac{\varepsilon_N^2}{\mu^\mu \varepsilon_D^2}\left(\frac{(\mu-1)\varepsilon_N^2}{\varepsilon_N^2 - \varepsilon_D^2}\right)^{\mu-1} - 1 \tag{8}$$

The constant k can be expressed as a function of ε_N or ε_D

$$k = \frac{\sum_{i=2}^{i=\mu} \binom{\mu}{i} \varepsilon_D^{2i}}{(\mu-1)\varepsilon_D^{2i} \sum_{i=1}^{i=\mu} \binom{\mu}{i} \varepsilon_D^{2i}} \tag{9}$$

All roots of equation (5) are real for $0 < \varepsilon_D < 1$ and $\mu > 2$, which provide the equiripple character of the magnitude function.

The positions of the minima $x_{\min}$ are not function of ε_N or ε_D. The positions of the maxima $x_{\max}$ change slightly with μ and ε_N or ε_D. Therefore, the shape of the magnitude is practically the same for any μ and ε_N or ε_D for $0 \le x \le 1$. Conversely, the magnitude function decreases $6\mu r$ dB/octave.

The equiriple rational function can be used as approximating function for filter sharpening because it is based on processing the date μ times with the same start-up filter (Chebyshev filter).

III. Design method

The simplest solution of filter sharpening is filtering the data k times with the existing filter. If the original filter's transfer function is $H(z)$, then the new transfer function is $(H(z))^k$. The new filter has the same band edges and the minimum attenuation, in decibels, is k times larger in the stopband. The drawback is that the passband ripple is also k times larger in decibels. This solution improves the stopband but degrades the passband.

The filter sharpening of analog filters [3] provides very low passband ripple and the Q factors of the dominant transfer-function pole pairs are reduced. An active RC filter cannot be efficiently implemented as a selective filter with very high pole-Q factors, while multiple low-Q factor SOS filters can be massively produced. Using the bilinear transformation, analog filters with low-Q factors are transformed into digital filters that are more robust for fixed point implementation [4]. Therefore, the same method as used in [3] can be used for digital filter sharpening.

The standard filter $H_s(z)$ should be chosen to be simple for implementation. For example, the standard filter can be the fourth-order Chebyshev filter that has small passband ripple (say A_s=0.1 dB) and it consists of two SOS filters

$$H_s(z) = \frac{1 + 2z^{-1} + z^{-2}}{1 + a_{s1}z^{-1} + a_{s2}z^{-2}} \frac{1 + 2z^{-1} + z^{-2}}{1 + a_{s3}z^{-1} + a_{s4}z^{-2}} \tag{10}$$

The coefficients are shown in Table 1.

The filter has a stopband attenuation of 8 dB at the stopband edge f_s=0.135; the passband edge frequency is f_p=0.1. The target filter, the cascaded connection of μ basic filters, should have the minimum stopband attenuation 40 dB; it also has large passband ripple 0.1•μ dB. To achieve a stopband attenuation of 40 dB, the number of cascades μ is equal to 5. Let us design the fourth-order Chebyshev filter $H_i(z)$ that has the passband ripple 0.5 dB for μ=5. The coefficients are shown in Table 1.

TABLE 1: COEFFICIENTS OF THE FILTER.

Coefficients	
a_{s1}	–1.26232
a_{s2}	0.44004
a_{s3}	–1.310268
a_{s4}	0.73834
a_{c1}	–1.46622
a_{c2}	0.58087
a_{c3}	–1.44789
a_{c4}	0.814144

The transfer function of the new filter $H(z)=(H_s(z))^\mu/H_i(z)$ can be simplified $H(z)=(H_s(z))^{\mu-1}H_c(z)$ in such a way that a new forth order filter becomes $H_c(z)= H_s(z)/H_i(z)$

$$H_c(z) = \frac{1 + a_{c1}z^{-1} + a_{c2}z^{-2}}{1 + a_{s1}z^{-1} + a_{s2}z^{-2}} \frac{1 + a_{c3}z^{-1} + a_{c4}z^{-2}}{1 + a_{s3}z^{-1} + a_{s4}z^{-2}} \tag{11}$$

The transfer function of cascaded connection of m standard and n compensation sections becomes

$$H(z) = (H_s(z))^m (H_c(z))^n \tag{12}$$

In Figures 1, 2 and 3, the attenuations of filters are shown for a number of combinations of m and n. The minimum stopband attenuation increases with the number of $H_s(z)$ sections while the passband ripple is the same as for the standard section, in this case 0.1 dB. The passband ripple is smaller than the ripple of the standard filter

$$A_n = 10n\log_{10}\left(\frac{1}{(\alpha-1)\alpha^{\mu-1}} \frac{(\mu-1)^{\mu-1}(\alpha^\mu - 1)^\mu}{\mu^\mu(\alpha^{\mu-1} - 1)^{\mu-1}}\right) \tag{13}$$

for

$$\alpha = 10^{A_s/10}, \; m = n(\mu - 1) \tag{14}$$

For example, for μ=5, n=1, A_s=0.1 dB, the input data is filtered 4 times using the standard filter $H_s(z)$ and ones using the compensation filter $H_c(z)$; the overall passband ripple is A_n=0.0058 dB and the minimum stopband attenuation is $A_{s\,\min}$=26 dB. For n=2, the data is filtered 8 times using $H_s(z)$ and twice using $H_c(z)$, the overall passband ripple is A_n=0.0116 dB and $A_{s\,\min}$=52 dB.

Attenuation of filters depends on the number of standard and compensation filters used. Solid lines represent transfer functions with passband ripple between 0dB and 0.1dB, dotted lines depict transfer functions with passband ripple between -0.1dB and 0dB, while dashed lines are transfer functions with passband ripple smaller than 0.1dB.

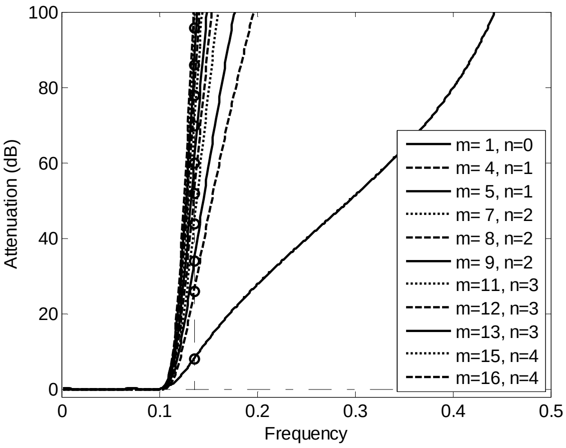

Fig. 1. Attenuation of filters for different number of standard and compensation filters.

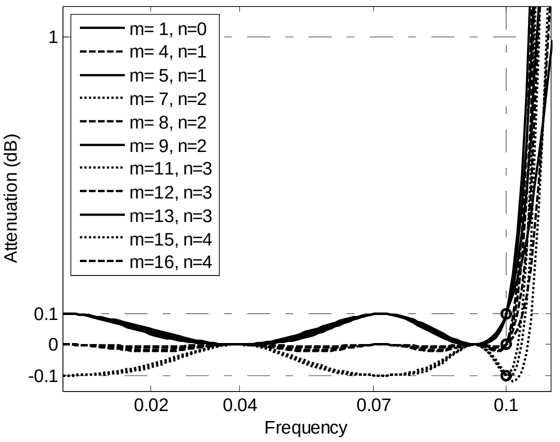

Fig. 2. Attenuation of filters in the passband.

Figure 4 demonstrates the implementation using field programmable gate array chips with only two time-folded fourth-order filters. Each filter consists of two cascaded SOS filters. Therefore, the two second-order sections are standard sections. The first SOS section has the transfer function zeros on the unit circle, while the second section has zeros within the unit circle. In fact, the control circuit is very simple, it consists of multiplexer and demultiplexer. It looks like some up-sampler or down-sampler in the classic multirate system, but it effectively works similarly to the classic time multiplexing system. Consequently, the timing diagram is very simple.

The term *small area* is used to emphasis that the used area is always the same and that it does not depend on the actual transfer function order. For example, with m=16 and n=4, the used are will be approximately 20 times larger. On the other hand, using the multiplier-less technique, the general multipliers can be implemented using only binary shifters, or a few adders and binary shifters, which can additionally reduce the used area on the FPGA chips.

The appropriate chip is selected according to the required functionality of the device, in which the implemented filter is used for filtering only.

In some applications it may be useful to use different filter specifications, with larger and smaller stopband attenuation, but with the same edge frequencies and very small passband ripple. Instead of implementing two filters, this filter can be used. Thus, the used area of one filter is smaller than the used area of two filters.

The benefits of using this method are (1) the usage of the same chip area regardless of the transfer function order, and (2) the possible savings in the number of general multipliers that are implemented using a small number of adders and binary shifters. More details on the multiplier-less technique can be found in [4].

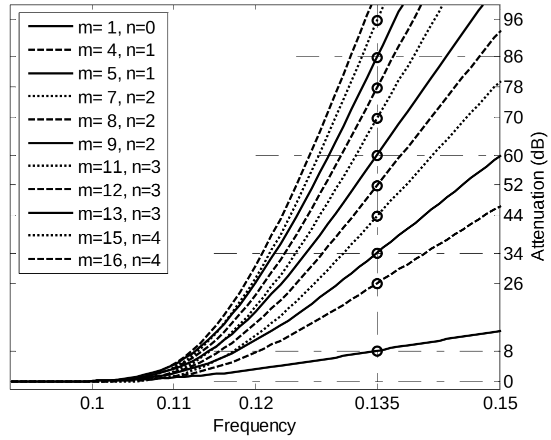

Fig. 3. Attenuation of filters in the transition.

IV. CONCLUSION

Using a small area of FPGA chips for placing only two fourth-order IIR filters, and hardware folding techniques to time-multiplex operations onto two filters, a number of different filters can be implemented. All filters have the same passband and stopband edges, the same or smaller passband ripples, and arbitrary minimum stopband attenuations that are functions of the down-sampling and up-sampling factor. The method called sharpened IIR filters is especially attractive for practicing engineers without extensive knowledge of filter theory, and for filter design without thorough and extensive analysis using filter design tools.

LITERATURA

[1] L. Narinder and B. Taylor, "Designing Control Circuits for High Performance DSP Systems," *Xilinx DSP Magazine*, pp. 55 57, 2005.

[2] M. Donadio, "Lost Knowledge Refound: Sharpened FIR Filters," *IEEE Signal Processing Magazine*, pp. 61-63, 2003.

[3] D. Rabrenović and M. Lutovac, "Low Q equiripple functions of nonminimum order," *Electronics Letters*, pp. 447-448, 1990.

[4] M. Lutovac, D. Tošić and B. Evans, *Filter Design for Signal Processing Using MATLAB and Mathematica.* Upper Saddle River, NJ: Prentice Hall, 2001.

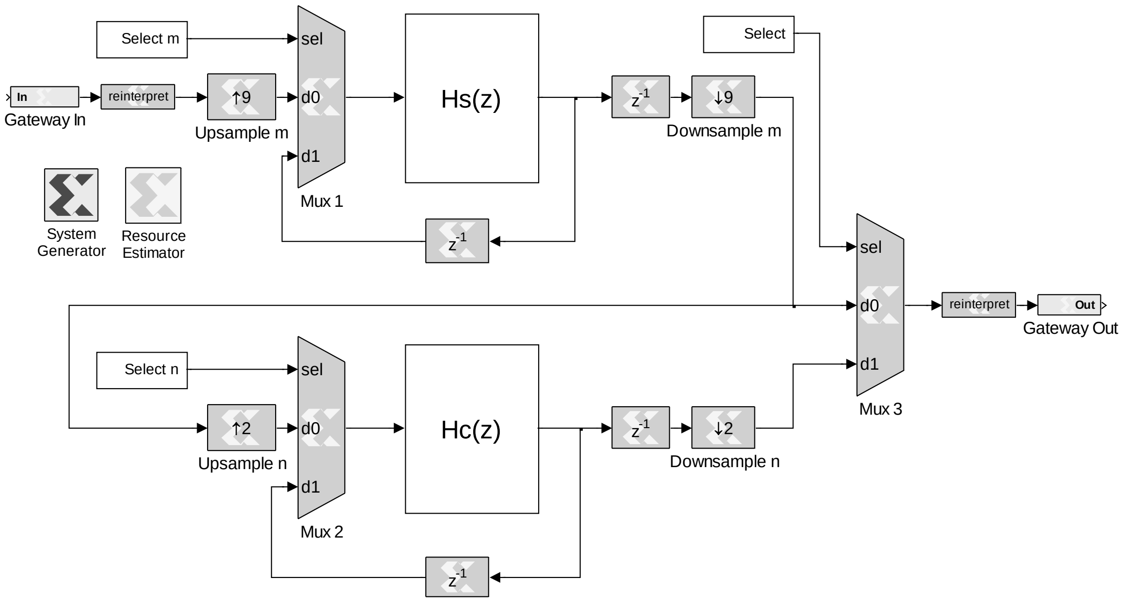

Fig. 4. Implementation of a sharpened IIR filter that consists of two filters *H*s(*z*) and *H*c(*z*), control circuits, two-input multiplexers and up-sample and down-sample blocks that control the data rate through the filters.