Frequency Filters Synthesis Based on the Signal-Flow Graphs

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***Abstract* —A technique of designing linear circuit structures using the M-C signal-flow graph theory is presented in this paper. First, the required form of the transfer function denominator is discussed. Subsequently, in a number of simple steps an M-C graph is designed. As an example a current-mode multifunction frequency filter with three second-generation current conveyors is presented. The properties of the proposed circuit enable to adjust the quality factor Q independently of the characteristic frequency f_0.**

***Keywords* — analogue signal processing, current conveyor, frequency filter, signal flow-graph**

I. Introduction

The current-mode frequency filters using current conveyors have recently drawn considerable attention mainly because of their bandwidth, linearity and dynamic range [1], [2]. Mostly the second-generation current conveyors or their modifications with one [3]-[5] or more current outputs [6]-[8] are used.

In despite of the number of already presented structures where the authors discuss solely the final circuit solution, the specific solution procedure is not clearly defined. For designing linear functional blocks a number of methods can be used: adjoint transformation [1], autonomous circuits [9], [10], high-order synthetic elements [11], or passive prototype [12].

Here, with advantage, for the frequency filter design the M-C signal-flow graph theory is used, even if it is more presented as a tool for analysis of already designed circuits. It is known that the transfer function of an M-C signal-flow graph can be determined using the equation, also labeled as Mason's gain formula [13]

$$K = \frac{Y}{X} = \frac{1}{\Delta}\sum_i P_i \Delta_i \,, \qquad (1)$$

where P_i is the transfer of the ith direct path from the input node X to the output node Y and Δ is the determinant of a graph that is given as follows

$$\Delta = V - \sum_k S_1^{(k)} V_1^{(k)} + \sum_l S_2^{(l)} V_2^{(l)} - \sum_m S_3^{(m)} V_3^{(m)} + \ldots, \qquad (2)$$

where V is product of the self-loops, $S_1^{(k)}$ is transfer of the kth oriented loop, and $V_1^{(k)}$ is product of all self-loops not-touching the kth oriented loop, $S_2^{(l)}$ is the transfer product of two not-touching oriented loops, and $V_2^{(l)}$ is product of the self-loops not-touching the lth oriented loops. If an oriented loop or kth direct path is touching all nodes then the product V or Δ_k is equal to 1. Δ_i is determinant of that part of the graph that is not-touching the ith direct path.

The presented design procedure starts with the description of the required form of the transfer function denominator, also known as the characteristic equation (CE). Using the knowledge of evaluating the determinant of an M-C graph (2), in several steps a corresponding M-C graphs can be defined, which fulfill the conditions of the required characteristic equation form.

A number of M-C graphs suitable for frequency filter design are proposed. As an example, a multifunction frequency filter is designed, where the quality factor Q can be changed independently of the characteristic frequency f_0.

II. Characteristic Equation

The determinant of an M-C graph (2) represents the characteristic equation (CE), which describes the behaviour of the analyzed circuit. If an nth-order frequency filter is designed, then the transfer function denominator (that is CE) must contain at least $n + 1$ terms, where all should be positive because of the stability. For a simple numerical design it is suitable for the number of terms to be minimal. Based on the chosen order of the filter the basic requirements for the M-C signal-flow graph (including active elements) that fulfill the feasibility conditions of a frequency filter with minimal number of both passive elements and characteristic equation terms can be described.

A transfer function of a second-order frequency filter can be generally described as a quotient of two polynomials

$$K(\boldsymbol{s}) = \frac{R(\boldsymbol{s})}{N(\boldsymbol{s})} = \frac{b_2\boldsymbol{s}^2 + b_1\boldsymbol{s} + b_0}{a_2\boldsymbol{s}^2 + a_1\boldsymbol{s} + a_0} \,. \qquad (3)$$

The numerator of the transfer function defines the frequency filter type (high-, low-, band-, all-pass, and band-stop). The denominator is valid and identical for all filter types and defines the parameters of the filter as the quality factor Q and characteristic frequency f_0

$$N(\boldsymbol{s}) = a_2\boldsymbol{s}^2 + a_1\boldsymbol{s} + a_0 = \boldsymbol{s}^2 + \frac{\omega_0}{Q}\boldsymbol{s} + \omega_0^2 \,, \qquad (4)$$

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where $\omega_0 = 2\pi f_0$. If a frequency filter is designed, where it should be possible to change the quality factor Q independently of the characteristic frequency f_0 by a passive element, the corresponding characteristic equation has to be as follows

$$\boldsymbol{CE} = \boldsymbol{s}^2 C_1 C_2 + \boldsymbol{s} C_1 G_3 + G_1 G_2 = 0 \tag{5}$$

or

$$\boldsymbol{CE} = \boldsymbol{s}^2 C_1 C_2 G_3 + \boldsymbol{s} C_1 G_1 G_2 + G_1 G_2 G_3 = 0\,. \tag{6}$$

Characteristic equations (5) and (6) respect the condition of minimal number of passive elements. In both cases, the quality factor can by adjusted via the conductance G_3

$$Q = \frac{\omega_0 C_2}{G_3}\,, \quad \text{or} \quad Q = \frac{\omega_0 C_2 G_3}{G_1 G_2}\,, \tag{7a,b}$$

where

$$\omega_0 = \sqrt{\frac{G_1 G_2}{C_1 C_2}}\,. \tag{8}$$

Hence, if a second-order frequency filter is designed, the following rules for a suitable M-C graph can be defined [14]:

D1: in the graph there exist a single oriented loop and two voltage nodes, while to one or both of them two or more passive elements are connected,

D2: in the graph there exist two mutually touching oriented loops and two voltage nodes, to which one passive element is connected,

D3: in the graph there exist one or more high-impedance nodes and three mutually touching oriented loops, which are touching the high-impedance nodes.

III. Frequency Filter Design

Since the M-C signal-flow graph method is used, it is necessary to know the corresponding M-C graph of the active element used. In the Fig. 1a, the electrical symbol of the generalized current conveyor (GCC) is shown [15]. Its M-C graph, which generally describes a group of second generation current conveyors CCII [16] is shown in Fig. 1b. The symbols Y_X, Y_Y, and Y_Z in Fig. 1b represent the sum of admittances connected to the corresponding active element port.

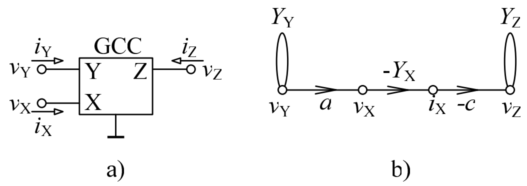


Fig. 1. a) Electrical symbol of GCC, b) reduced M-C graph of general CCII.

The relationship between the port voltages and currents of the active element used can be described by following hybrid matrix

$$\begin{bmatrix} v_X \\ i_Y \\ i_Z \end{bmatrix} = \begin{bmatrix} 0 & a & 0 \\ 0 & 0 & 0 \\ c & 0 & 0 \end{bmatrix} \begin{bmatrix} i_X \\ v_Y \\ v_Z \end{bmatrix}, \tag{9}$$

where a is the voltage transfer coefficient and c is the current transfer coefficient, which can be of value ±1.

The design procedure of an M-C graph, that characteristic equation fulfills the condition (5), using rule **D2** is shown in Fig. 2.

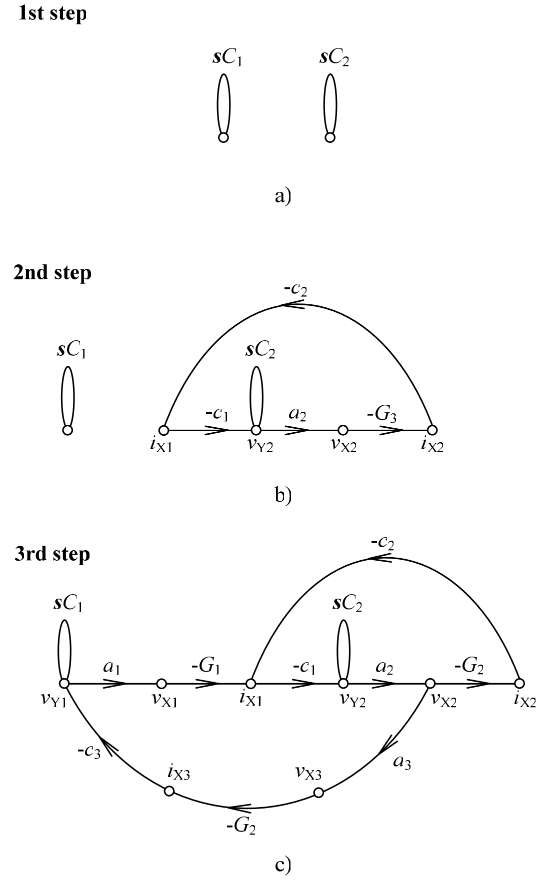


Fig. 2. Designing procedure of an M-C graph with current conveyors using **D2**.

The determinant of the graph in single steps according to Fig. 2a to Fig. 2c is

$$\Delta = \boldsymbol{s}^2 C_1 C_2\,, \tag{10a}$$

$$\Delta = \boldsymbol{s}^2 C_1 C_2 + c_1 a_2 c_2 \boldsymbol{s} C_1 G_3\,, \tag{10b}$$

$$\Delta = \boldsymbol{CE} = \boldsymbol{s}^2 C_1 C_2 + c_1 a_2 c_2 \boldsymbol{s} C_1 G_3 + a_1 c_1 a_3 c_3 G_1 G_2 = 0\,. \tag{10c}$$

In the 3rd step, the determinant already generally agrees with the required form of the characteristic equation (5). Similarly, a group of M-C graphs shown in Fig. 3 has been designed.

IV. Simulations

Further, the behaviour of the structure according the M-C graph in Fig. 3f will be analyzed. In Fig. 4, the flow-graph of a frequency filter working in current-mode is shown. If it is valid: $a_1 = a_2 = a_3 = 1$, $c_{12} = c_{31} = -1$, and $c_{11} = c_{21} = c_{22} = c_{32} = 1$, then in the current-mode the proposed circuit works as a high-pass filter

$$\boldsymbol{K}_{HP} = \frac{\boldsymbol{I}_{HP}}{\boldsymbol{I}_{IN}} = \frac{\boldsymbol{s}^2 C_1 C_2}{\boldsymbol{CE}}\,, \tag{11}$$

low-pass filter

$$\boldsymbol{K}_{LP} = \frac{\boldsymbol{I}_{LP}}{\boldsymbol{I}_{IN}} = \frac{G_1 G_2}{\boldsymbol{CE}}\,, \tag{12}$$

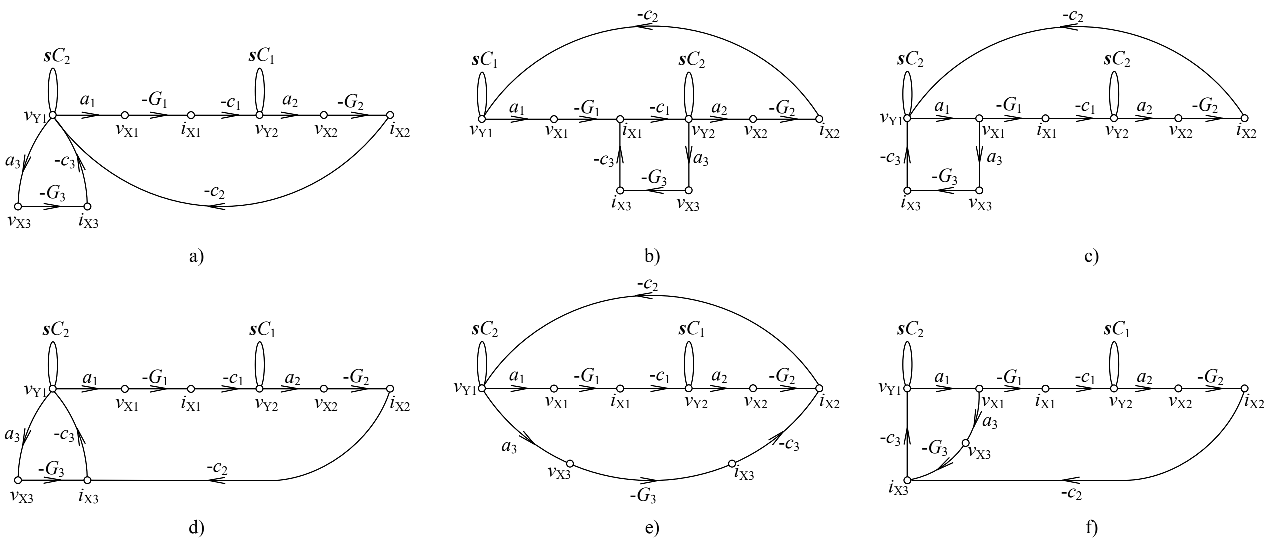


Fig. 3. The M-C graphs with current conveyors using **D2**.

and band-pass filter

$$\boldsymbol{K}_{\text{BP}} = \frac{\boldsymbol{I}_{\text{BP}}}{\boldsymbol{I}_{\text{IN}}} = -\frac{\boldsymbol{s}C_1G_1}{\boldsymbol{CE}}, \tag{13}$$

where $\boldsymbol{CE}$ is (5).

Adding up the currents $\boldsymbol{I}_{\text{HP}}$ and $\boldsymbol{I}_{\text{LP}}$ a band-stop filter (notch) can be realized

$$\boldsymbol{K}_{\text{BS}} = \frac{\boldsymbol{I}_{\text{HP}} + \boldsymbol{I}_{\text{LP}}}{\boldsymbol{I}_{\text{IN}}} = \frac{\boldsymbol{s}^2C_1C_2 + G_1G_2}{\boldsymbol{CE}}. \tag{14}$$

Similarly, an all-pass filter can be realized by adding up the output currents $\boldsymbol{I}_{\text{HP}}$, $\boldsymbol{I}_{\text{LP}}$, and $\boldsymbol{I}_{\text{BP}}$, while $G_1 = G_3$

$$\boldsymbol{K}_{\text{AP}} = \frac{\boldsymbol{I}_{\text{HP}} + \boldsymbol{I}_{\text{BP}} + \boldsymbol{I}_{\text{LP}}}{\boldsymbol{I}_{\text{IN}}} = \frac{\boldsymbol{s}^2C_1C_2 - \boldsymbol{s}C_1G_1 + G_1G_2}{\boldsymbol{CE}}. \tag{15}$$

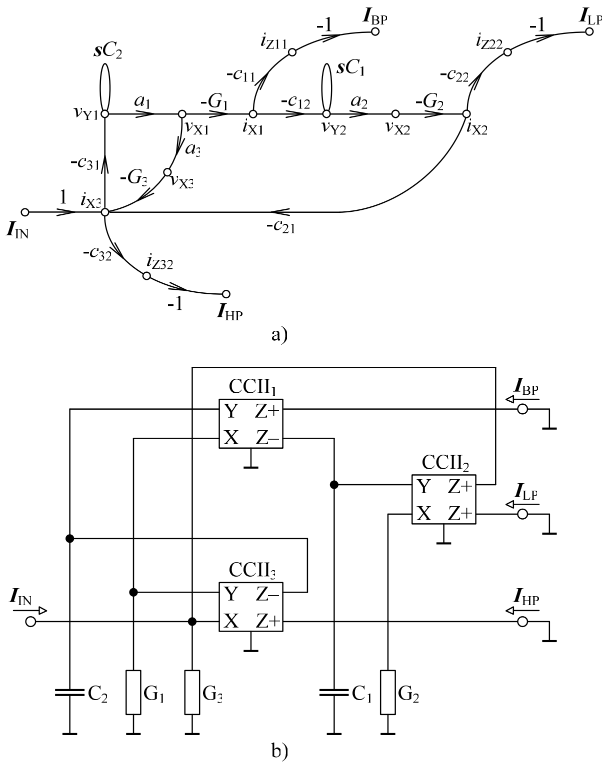


Fig. 4. a) The M-C graph of the current-mode filter, b) its circuit solution.

The behaviour of the analyzed multifunction filter was further simulated in OrCAD. For given values of the characteristic frequency $f_0 = 1$ MHz, and capacitors $C_1 = C_2 = 150$ pF using (8) the conductances are $G_1 = G_2 = 0.94$ mS. For chosen values 0.5, 1, and 10 of the quality factor Q, values of the conductor G_3 must be 1.88 mS, 0.94 mS, and 0.09 mS. All active elements have been defined by the third-level model of the universal current conveyor UCC-N1B [17]. The simulation results of the current-mode multifunction filter are shown in Fig. 5 and Fig. 6.

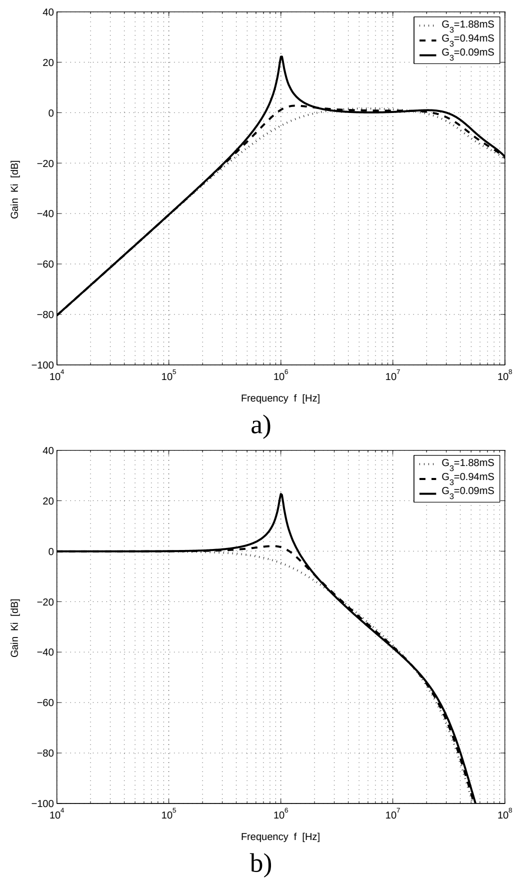
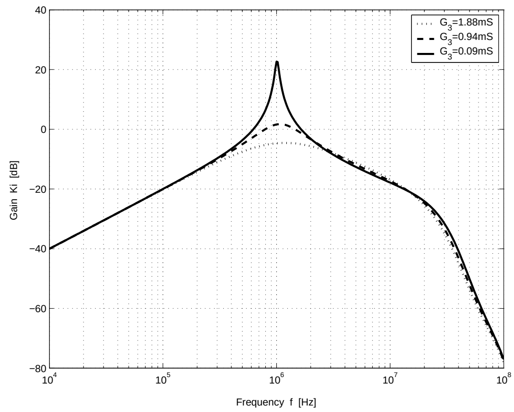
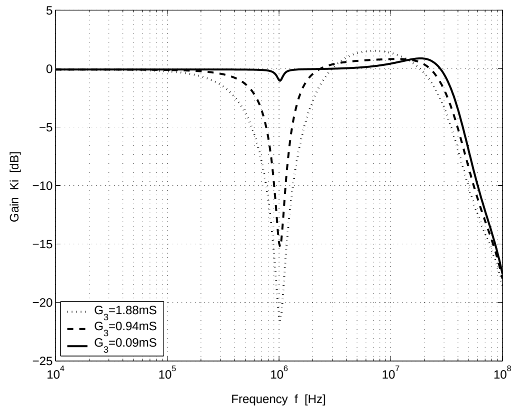


Fig. 5. Simulation results of the a) high-, b) low-pass filter working in the current-mode.



a)



b)

Fig. 6. Simulation results of the a) band-pass, b) band-stop filter working in current-mode.

The simulation results of the analyzed multifunction frequency filter are very satisfactory. The usage of three active elements enables to achieve higher values of the quality factor Q as shown in [18]. In the frequency area of 20 MHz, the parasitic properties of the active elements used start to be significant. In case of the high-pass and band-stop filter the magnitude decreases its value. However, in case of the low- and band-pass this property is not unsuitable.

V. Conclusion

This report deals with the usage of the M-C signal-flow graphs for the design of frequency filters with required form of the characteristic equation. Based on the presented theoretical results, a number of M-C graphs with current conveyors have been defined that lead to the design of frequency filters, where the quality factor Q can be changed independently of the characteristic frequency f_0. Since a minimal number of passive and active elements is used, low passive and active sensitivities can be expected. On the selected M-C graph further design procedure of a multifunction frequency filter working in current-mode has been presented. The behaviour of the proposed filter has been verified by OrCAD simulations.

References

[1] G. W. Roberts, and A. S. Sedra, "All Current-Mode Frequency Selective Circuits," *Electronics Letters*, vol. 25, no. 12, pp. 759-760, June 1989.

[2] S. Takagi, "Analog Circuit Designs in the Last Decade and Their Trends toward the 21st Century," *IEICE Trans. Fundamentals*, vol. E84-A, no. 1, pp. 68-79, Jan. 2001.

[3] S. Minaei, O. Cicekoglu, H. Kuntman, and S. Türköz, "High output impedance current-mode lowpass, bandpass and highpass filters using current controlled conveyors," *Int. J. Electronics*, vol. 88, no. 8, pp. 915-922, 2001.

[4] S. Minaei, O. K. Sayin, and H. Kuntman, "A New CMOS Electronically Tunable Current Conveyor and Its Application to Current-Mode Filters," *IEEE Trans. Circuits Systems I*, vol. 53, No. 7, pp. 1448-1457, July 2006.

[5] S. Minaei, and S. Türköz, "Current-Mode Electronically Tunable Universal Filter Using Only Plus-Type Current Controlled Conveyors and Grounded Capacitors," *ETRI Journal*, vol. 26, no. 4, Aug. 2004.

[6] R. Senani, V. K. Singh, A. K. Singh, and D. R. Bhaskar, "Tunable Current-Mode Universal Biquads employing only three MOCCs and all grounded passive elements: Additional New Realizations," *Frequenz*, vol. 59, no. 9-10, 2005.

[7] E. Yuce, S. Minaei, and O. Cicekoglu, "Resistorless floating immittance function simulators employing current controlled conveyors and a grounded capacitor," *Electrical Engineering*, vol. 88, no. 6, pp. 549-525, Dec. 2005.

[8] N. Pandey, S. K. Paul, A. Bhattacharyya, and S. B Jain, "A novel current controlled current mode universal filter: SITO approach," *IEICE Electronics Express*, vol. 2, no. 17, pp. 451-457, Sep. 2005.

[9] C. Hou, Y. Wu, S. Liu, "New configuration for single-CCII first-order and biquadratic current mode filters," *Int. J. Electronics*, vol. 71, no. 4, pp. 637-644, 1991.

[10] K. Vrba, J. Cajka, V. Zeman, "New RC-Active Network Using Current Conveyors," *Radioengineering*, vol. 6, no. 2, pp. 18-21, 1997.

[11] R. Sponar, "High-Order Imittance Synthetic Oneport Elements in Frequency Filters with Current Conveyors," *Elektrorevue – Internet Journal*, no. 13, 2004.

[12] D. Biolek, T. Gubek, V. Biolkova, "Optimization of CDTA–based circuits simulating ladder structures," *Proc. WSEAS Transactions on Mathematics*, pp. 783-788, 2004.

[13] D Biolek, *Solving electronics circuits*, Prague, BEN, 2004 (in Czech).

[14] J. Koton, P.A. Ushakov, "Designing Frequency Filters Using the Signal-Flow Graph Theory," In *First forum of young researches.* Izhevsk: Publishing House of ISTU, 2008. pp. 297-304.

[15] J. Cajka, T. Dostal, K. Vrba, "General view on current conveyors," *Int. J. of Circuit Theory and Applications*, vol. 32, no. 3, pp. 133-138, 2004.

[16] J. Koton, M. Minarcik, "The Usage of Signal Flow Graphs for Analysis of Circuits with Current Conveyors," *Elektrorevue – Internet Journal*, no. 39, 2006.

[17] R. Sponar, K. Vrba, "Measurements and Behavioral Modeling of Modern Conveyors," *IJCSNS Int. J. Comp. Science Network Sec.*, vol. 6, no. 3A, pp. 57-65, March 2006.

[18] J. Koton, K. Vrba, "New Multifunctional Frequency Filter Working in Current-mode," *Mobile and Wireless Communication Networks*, vol. 2007, no. 9, pp. 569-577, 2007.