Probability Density Function of M-ary FSK Signal in the Presence of Gaussian Noise, Intersymbol Interference and Weibull Fading

Dragana S. Krstić, Petar Nikolić, Goran Lj. Stamenović, and Mihajlo Č. Stefanović

Abstract — **In this paper the receiver for demodulation of M-FSK signals in the presence of Gaussian noise, intersymbol interference and Weibull fading will be considered. Probability density function (PDF) of M-ary FSK signals in the presence of noise, interference and fading will be derived.**

Keywords — **Gaussian noise, Intersymbol Interference, M-ary Frequency Shift Keying, Probability Density Function, Weibull Fading**

I. Introduction

THE performances of communication systems can be seriously disturbed by the influence of Gaussian noise, intersymbol interference and Weibull fading. In order to view the influence of Gaussian noise, intersymbol interference and Weibull fading on the performances of an *M*-ary FSK system we will derive the probability density function of *M*-ary FSK receiver output signal and joint probability density function of the output signal and its derivative.

Noise, interference and fading processes can seriously degrade the performance of communication systems [1], [2]. In the paper [3], the performance evaluation of several types of FSK and CPFSK receivers was investigated in detail. Also, the error probability of the cross-correlator receiver for binary digital frequency modulation detection is studied using theoretical analysis and computer simulations [4]. Performance analysis of system with selection combining over correlated Weibull fading channels in the presence of cochannel interference is given in [5]. In [6] probability density function of *M*-ary FSK signal in the presence of Gaussian noise, intersymbol interference and Rayleigh fading is calculated.

In this paper the receiver for demodulation of *M*-FSK signals in the presence of Gaussian noise, intersymbol interference and Weibull fading will be considered. Probability density function (PDF) of *M*-ary FSK receiver output signals in the presence of these interferences will be derived and obtained numerical results will be shown graphically for different parameters values.

Dragana S. Krstić is with the Faculty of Electronic Engineering, Niš, Serbia, Aleksandra Medvedeva 14 (phone: +381631045808; fax: +38118588399; e-mail: dragana@telfak.ni.ac.yu).

Petar Nikolić is with Tigar Tyres, Pirot, Serbia

Goran Lj. Stamenović is with Tigar, Pirot, Serbia

Mihajlo Č. Stefanović is with the Faculty of Electronic Engineering, Niš, Serbia

II. System Model

We will consider in this paper the model of *M*-ary FSK system shown at Fig. 1.

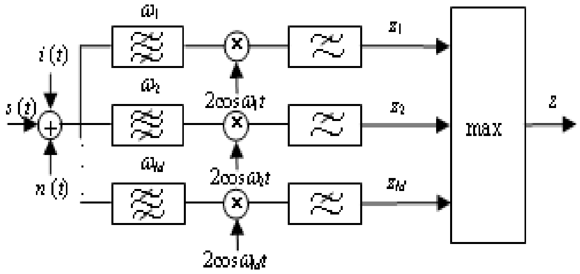

Fig.1. System model for coherent demodulation of *M*-ary FSK signal

This system has *M* branches. Each branch consists of the bandpass filter and correlator. The correlator is consisting of multiplier and lowpass filter. The signal at the receiver input is digital frequently modulated signal corrupted by additive Gaussian noise, intersymbol interference and Weibull fading.

Transmitted signal for the hypothesis H_i is:

$$s(t) = A\cos\omega_i t \tag{1}$$

where A denotes the amplitude of modulated signal and has Weibull distribution:

$$p_A(A) = \frac{\beta}{\Omega}\left(\frac{A}{\Omega}\right)^{\beta-1} \exp\left[\left(-\frac{A}{\Omega}\right)^{\beta}\right], \quad A \ge 0 \tag{2}$$

Gaussian noise at the receiver input is given with:

$$n(t) = \sum_{i=1}^{M} x_i \cos\omega_i t + y_i \sin\omega_i t, \quad i=1, 2,...M \tag{3}$$

where x_i and y_i are components of Gaussian noise, with zero means and variances σ^2.

The interference $i(t)$ can be written as:

$$i(t)=\sum_{i=1}^{M} A_i \cos(\omega_i t+\theta_i) \qquad (4)$$

where phases θ_i have uniform probability density function.

These signals pass first through bandpass filters whose central frequencies $\omega_1, \omega_2,..., \omega_M$ correspond to hypotheses $H_1,H_2,...,H_M$. After multiplying with signal from the local oscillator, they pass through lowpass filter. The filter cuts all spectral components which frequencies are greater than the border frequency of the filter.

If $z_1, z_2, ... z_M$ are receiver branches output signals, then the M-FSK receiver output signal is:

$$z=\max\{z_1, z_2 ... z_M\} \qquad (5)$$

The probability density of output signal is:

$$p(z)=\sum_{i=1}^{M} p_{z_i}(z)\cdot \prod_{\substack{j=1\\ j\neq i}}^{M} F_{z_j}(z) \qquad (6)$$

In the case of the hypothesis H_1, transmitted signal is:

$$s(t)=A\cos\omega_1 t \qquad (7)$$

while the receiver branch output signals are:

$$z_1=A+x_1+A_1\cos\theta_1 \qquad (8)$$

…

$$z_k=x_k+A_k\cos\theta_k \text{, } k=2,3,...,M \qquad (9)$$

It is necessary to define the probability density functions of the branches output signals and cumulative probability density of these signals to obtain probability density function of M-ary FSK receiver output signal.

The conditional probability density functions for the signals $z_1, z_2,..., z_M$ are:

$$p_{z_1/A,\theta_1}(z_1)=\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z_1-A-A_1\cos\theta_1)^2}{2\sigma^2}} \qquad (10)$$

$$p_{z_k/A,\theta_k}(z_k)=\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z_k-A_k\cos\theta_k)^2}{2\sigma^2}},$$

$$k=2,3,...,M \qquad (11)$$

By averaging (10) and (11) we obtain the probability density functions of the branches output signals:

$$p_{z_1}(z_1)=\int_0^{\infty}\int_{-\pi}^{\pi} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z_1-A-A_1\cos\theta_1)^2}{2\sigma^2}}\cdot$$

$$\cdot\frac{\beta}{\Omega}\left(\frac{A}{\Omega}\right)^{\beta-1} \exp\left[\left(-\frac{A}{\Omega}\right)^{\beta}\right] dA\frac{1}{2\pi}d\theta_1 \qquad (12)$$

…

$$p_{z_k}(z_k)=\int_0^{\infty}\int_{-\pi}^{\pi} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z_k-A_k\cos\theta_k)^2}{2\sigma^2}}\cdot$$

$$\cdot\frac{\beta}{\Omega}\left(\frac{A}{\Omega}\right)^{\beta-1} \exp\left[\left(-\frac{A}{\Omega}\right)^{\beta}\right] dA\frac{1}{2\pi}d\theta_k \qquad (13)$$

The cumulative distributions of the signals $z_1,z_2,..., z_M$ are:

$$F_{z_1}(z_1)=\int_{-\infty}^{z_1}\int_0^{\infty}\int_{-\pi}^{\pi} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z_1-A-A_1\cos\theta_1)^2}{2\sigma^2}}\cdot$$

$$\cdot\frac{\beta}{\Omega}\left(\frac{A}{\Omega}\right)^{\beta-1} \exp\left[\left(-\frac{A}{\Omega}\right)^{\beta}\right] dA\frac{1}{2\pi}d\theta_1 dz_1 \qquad (14)$$

…

$$F_{z_k}(z_k)=\int_{-\infty}^{z_k}\int_0^{\infty}\int_{-\pi}^{\pi} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z_k-A_k\cos\theta_k)^2}{2\sigma^2}}\cdot$$

$$\cdot\frac{\beta}{\Omega}\left(\frac{A}{\Omega}\right)^{\beta-1} \exp\left[\left(-\frac{A}{\Omega}\right)^{\beta}\right] dA\frac{1}{2\pi}d\theta_k dz_k \qquad (15)$$

The probability density function of M-ary FSK receiver output signal can be obtained from:

$$p(z)=\sum_{i=1}^{M} p_{z_i}(z)\cdot \prod_{\substack{j=1\\ j\neq i}}^{M} F_{z_j}(z) \qquad (16)$$

The probability density function of M-ary FSK receiver output signal in the case of hypothesis H_1 is:

$$p_{z_1}(z_1)=\sum_{i=1}^{M} p_{z_{1i}}(z_1)\cdot \prod_{\substack{j=1\\ j\neq i}}^{M} F_{z_{1j}}(z_1) \qquad (17)$$

The joint probability density function of the output signal z and its derivative is:

$$p_{z\dot{z}}(z,\dot{z})=\sum_{i=1}^{M} p_{z_i\dot{z}_i}(z,\dot{z})\cdot \prod_{\substack{j=1\\ j\neq i}}^{M} F_{z_j}(z) \qquad (18)$$

III. Numerical Results

In many practical telecommunication systems dual branch FSK receiver is used. We now consider this system because of its easy implementation and very good performances.

The probability density function, in the case of dual branch, has a form:

$$p(z)=p_{z_1}(z)\cdot F_{z_2}(z)+p_{z_2}(z)\cdot F_{z_1}(z) \qquad (19)$$

The probability density functions $p(z)$, for various values of the parameters A_i, Ω, β and σ are given at Figs. 2. to 9.

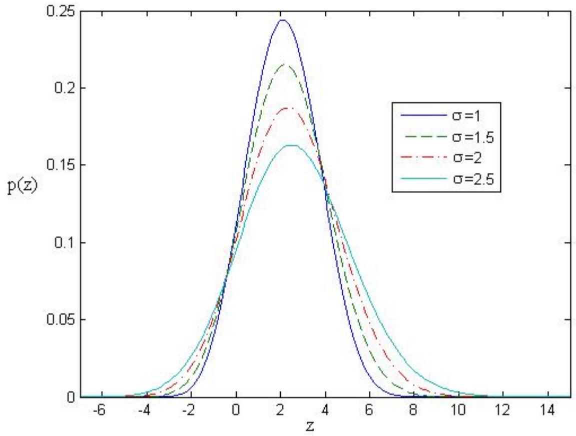

Fig.2. The probability density functions *p(z)* for the parameters $A_i=2, \sigma=1,1.5,2,2.5, \Omega=2$ and $\beta=3$

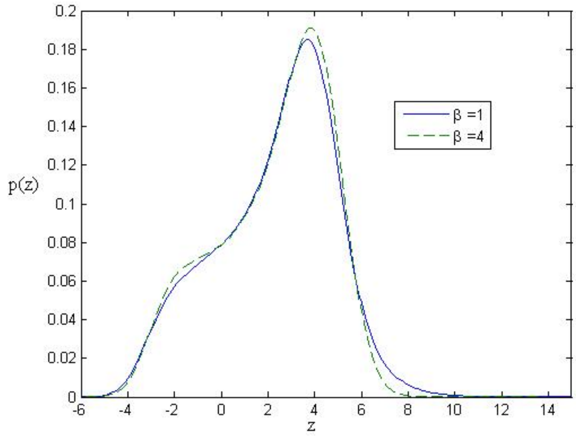

Fig.5. The probability density functions *p(z)* for the parameters $A_i=4, \sigma=1, \Omega=1$ and $\beta=1,4$

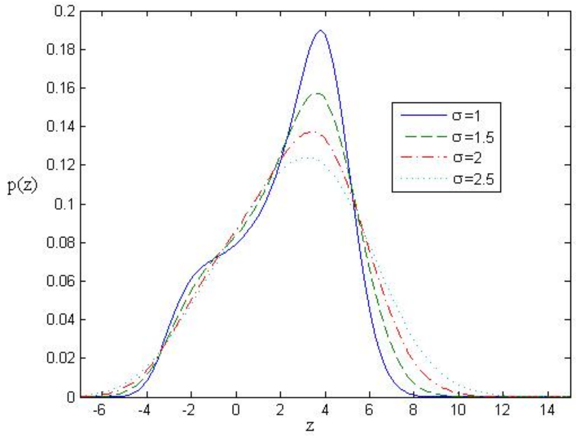

Fig.3. The probability density functions *p(z)* for the parameters $A_i=4, \sigma=1, 1.5,2,2.5, \Omega=1$ and $\beta=2$

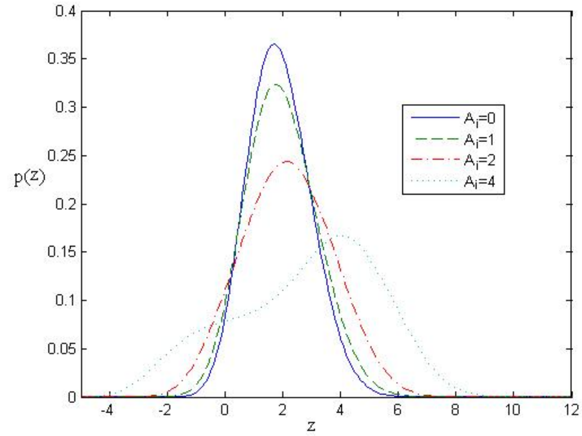

Fig.6. The probability density functions *p(z)* for the parameters $A_i=0,1,2,4, \sigma=1, \Omega=2$ and $\beta=3$

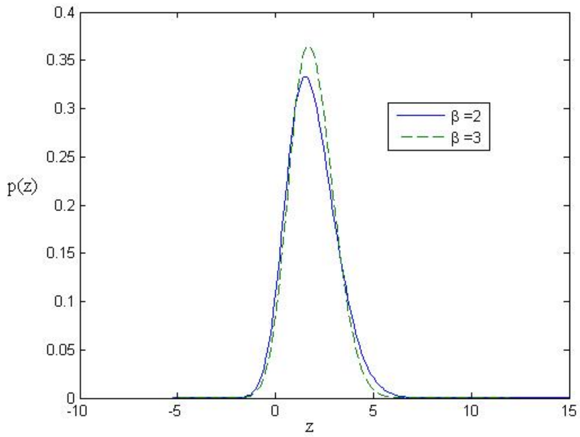

Fig.4. The probability density functions *p(z)* for the parameters $A_i=0, \sigma=1, \Omega=2$ and $\beta=2,3$

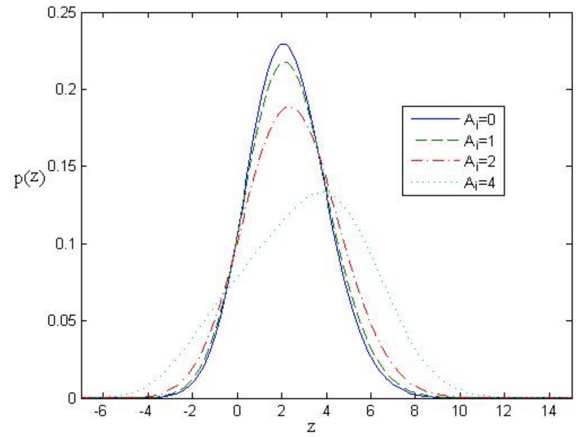

Fig.7. The probability density functions *p(z)* for the parameters $A_i=0,1,2,4, \sigma=2, \Omega=1$ and $\beta=4$

We can see from these figures that graphs become more distorted for bigger interference amplitude, whatever fading and noise parameters grow or reduce. Therefore, the influence of intersymbol interference for small signals is the most critical.

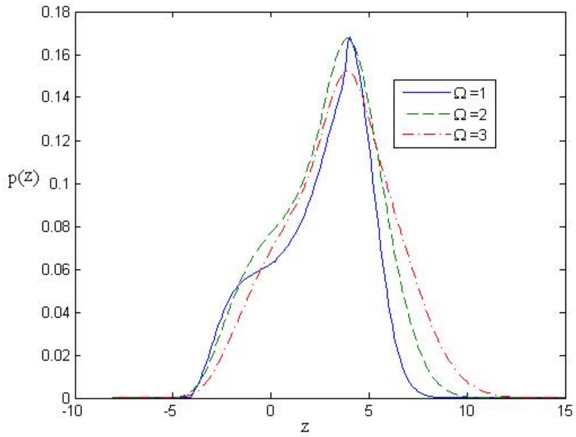

Fig.8. The probability density functions *p(z)* for the parameters $A_i=4$, $\sigma=1$, $\Omega=1,2,3$ and $\beta=2$

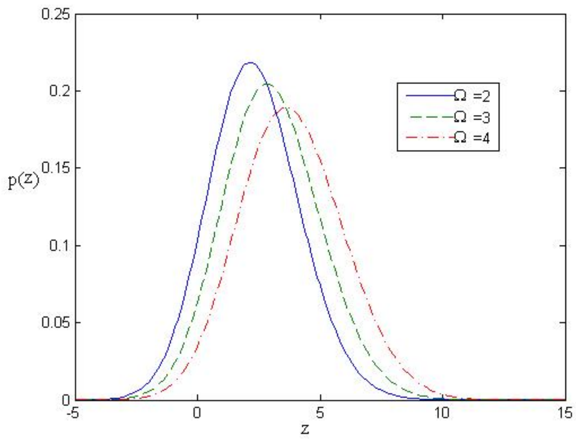

Fig.9. The probability density functions *p(z)* for the parameters $A_i=1$, $\sigma=2$, $\Omega=2,3,4$ and $\beta=5$

IV. CONCLUSION

In this paper the statistical characteristics of the receiver output signal for coherent *M*-FSK signal demodulation are derived. The receiver input signal is corrupted by additive Gaussian noise, intersymbol interference and Weibull fading.

In this paper the probability density function of *M*-ary FSK receiver output signal is calculated and probability density function curves of dual branch FSK receiver output signal are presented for various parameters values.

The bit error probability, the signal error probability and the outage probability can be determined by the probability density function of an output signal. Also, the moment generating function, the cumulative distribution of output signals and moment and variance of output signals can be derived by probability density function of the output signals. The average level crossing rate and average fade duration of output signal process can be calculated by the joint probability density of output signal and its derivative. An expression for the calculation of autocorrelation function can be derived by the joint probability density function of the output signal at two time instants. The use of the Winner-Hinchine theorem gives us the spectral power density function of output *M*-ary FSK signal. Also, by this function likelihood function of *M*-FSK system when the decision is done by two samples can be calculated. The joint probability density function of the output signal at two times instants is important for the case when the noise is correlated.

REFERENCES

[1] J. G. Proakis, *Digital Communications*, 2nd ed. New York, McGraw-Hill, 1989.

[2] Marvin K. Simon, Mohamed-Slim Alouni, *Digital Communication over Fading Channels*, Second Edition, Wiley Interscience, New Jersey, 2005, pp. 586.

[3] Hao Miin-Jong and B. Stephen Wicker, "Performance evaluation of FSK and CPFSK optical communication systems: a stable and accurate method", *J. Lightwave Technol.*, Vol. 13, No. 8, Aug. 1995.

[4] K. A. Farrell and P. J. McLane, "Performance of the cross-correlator receiver for binary digital frequency modulation", *IEEE Trans. Commun.*, Vol. 45, No 5, May 1997.

[5] Stefanović M, et al. "Performance analysis of system with selection combining over correlated Weibull fading channels in the presence of cochannel interference", *Int J Electron Commun (AEU)*, doi: 10.1016/j.aeue.2007.09.006, 2007.

[6] Dragana Krstić, Mihajlo Stefanović, Natasa Kapacinović, Srdjan Jovković, Dušan Stefanović, "Probability Density Function of M-ary FSK Signal in the Presence of Noise, Interference and Fading", *WSEAS TRANSACTIONS on COMMUNICATIONS*, ISSN: 1991-8763, Issue 4, Volume 3, April 2008, pp. 438-447.